

Neutrino masses in the economical 3-3-1 model

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We show that, in frameworks of the economical 3-3-1 model, the suitable pattern of neutrino masses arises from the three quite different sources - the lepton-number conserving, the spontaneous lepton-number breaking and the explicit lepton-number violating, widely ranging over the mass scales including the GUT one: $u \sim O(1)$ GeV, $v \approx 246$ GeV, $\omega \sim O(1)$ TeV and $M \sim O(10^{16})$ GeV. At the tree-level, the model contains three Dirac neutrinos: one massless, two large with degenerate masses in the order of the electron mass. At the one-loop level, the left-handed and right-handed neutrinos obtain Majorana masses $M_{L,R}$ in orders of $10^{-2} - 10^{-3}$ eV and degenerate in $M_R = -M_L$, while the Dirac masses get a large reduction down to eV scale through a finite mass renormalization. In this model, the contributions of new physics are strongly signified, the degenerations in the masses and the last hierarchy between the Majorana and Dirac masses can be completely removed by heavy particles. All the neutrinos get mass and can fit the data.

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I. INTRODUCTION

Until now the unique evidence supported physics beyond the standard model (SM) is nonzero neutrino masses through their oscillation. The recent experimental results of SuperKamiokande Collaboration [1], KamLAND [2] and SNO [3] confirm that neutrinos have tiny masses and oscillate. This implies that the standard model must be extended. Among the beyond-SM extensions, the models based on the $SU(3)_C \otimes SU(3)_L \otimes U(1)_X$ (3-3-1) gauge group [4, 5] have some intriguing features: First, they can give partial explanation of the generation number problem. Second, the third quark generation has to be different from the first two, so this leads to the possible explanation of why top quark is uncharacteristically heavy.

In one of the 3-3-1 models, three lepton triplets are of the form (ν_L, l_L, ν_R^c) , where the third members are related to the right-handed (RH) components of neutrino fields ν . The scalar sector in this model is minimal with just two Higgs triplets, hence it has been called the economical 3-3-1 model [6, 7]. The general Higgs sector is very simple and consists of three physical scalars (two neutral and one charged) and eight Goldstone bosons - the needed number for massive gauge bosons [8]. The model is consistent and possesses the key properties: (i) There are three quite different scales of the vacuum expectation values (VEVs): $u \sim O(1)$ GeV, $v \approx 246$ GeV, and $\omega \sim O(1)$ TeV; (ii) There exist two types of the Yukawa couplings with very different strengths: the lepton-number conserving (LNC) h 's and the lepton-number violating (LNV) s 's satisfying $s \ll h$. The resulting model yields interesting physical phenomenologies due to mixings in the Higgs [8], gauge [6, 9] and quark [10] sector.

At the tree level, the neutrino spectrum in this model contains three Dirac fields with one massless and two degenerate in mass $\sim h^\nu v$, where the Majorana fields ν_L and ν_R are massless. This spectrum is not realistic under the data because there is only one squared-mass splitting. Moreover, since the observed neutrino masses are so small, the Dirac mass is unnatural, and then one must understand what physics gives $h^\nu v \ll h^l v$ - the mass of charged leptons.

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In contrast to usual cases [11], in which the problem can be solved, in this model the neutrinos including RH ones can only get small masses through radiative corrections.

The aim of this work is carrying out radiative corrections for the neutrino masses and gives a possible explanation of why the neutrino Dirac masses are small. This is not the result of a seesaw [11], however, is due to a finite mass renormalization arising from a very different radiative mechanism. We will show that the neutrinos can get mass not only from the standard symmetry breakdown, but also from the electroweak $SU(3)_L \otimes U(1)_X$ breaking associated with spontaneous lepton-number breaking (SLB), and even through the explicit lepton-number violating processes of a new physics.

At the one-loop level the Dirac neutrinos can get a large reduction in mass, the fields ν_L and ν_R obtain Majorana masses $M_{L,R}$. The degeneration of $M_R = -M_L$ is removed by heavy particles. The total mass spectrum for the neutrinos are therefore neat and can fit the data.

The rest of this paper is organized as follows: In Section II, we give a brief review of the economical 3-3-1 model, the mass mechanisms of neutrinos are represented. Section III devotes detailed calculations and analysis of the neutrino mass spectrum. We summarize our results and make conclusions in the last section - Sec. IV.

II. A REVIEW OF THE ECONOMICAL 3-3-1 MODEL

The particle content in this model, which is anomaly free, is given as follows

$$\begin{aligned} \psi_{aL} &= (\nu_{aL}, l_{aL}, (\nu_{aR})^c)^T \sim (3, -1/3), \quad l_{aR} \sim (1, -1), \quad a = 1, 2, 3, \\ Q_{1L} &= (u_{1L}, d_{1L}, U_L)^T \sim (3, 1/3), \quad Q_{\alpha L} = (d_{\alpha L}, -u_{\alpha L}, D_{\alpha L})^T \sim (3^*, 0), \quad \alpha = 2, 3, \\ u_{aR} &\sim (1, 2/3), \quad d_{aR} \sim (1, -1/3), \quad U_R \sim (1, 2/3), \quad D_{\alpha R} \sim (1, -1/3), \end{aligned} \quad (1)$$

where the values in the parentheses denote quantum numbers based on the $(SU(3)_L, U(1)_X)$ symmetry. The electric charge operator in this case takes a form

$$Q = T_3 - \frac{1}{\sqrt{3}}T_8 + X, \quad (2)$$

where T_i ($i = 1, 2, \dots, 8$) and X , respectively, stand for $SU(3)_L$ and $U(1)_X$ charges. The electric charges of the exotic quarks U and D_α are the same as of the usual quarks, i.e., $q_U = 2/3$, $q_{D_\alpha} = -1/3$.

The spontaneous symmetry breaking in this model is obtained by two stages:

$$SU(3)_L \otimes U(1)_X \rightarrow SU(2)_L \otimes U(1)_Y \rightarrow U(1)_Q. \quad (3)$$

The first stage is achieved by a Higgs scalar triplet with a VEV given by

$$\chi = (\chi_1^0, \chi_2^-, \chi_3^0)^T \sim (3, -1/3), \quad \langle \chi \rangle = \frac{1}{\sqrt{2}}(u, 0, \omega)^T. \quad (4)$$

The last stage is achieved by another Higgs scalar triplet needed with the VEV as follows

$$\phi = (\phi_1^+, \phi_2^0, \phi_3^+)^T \sim (3, 2/3) \quad \langle \phi \rangle = \frac{1}{\sqrt{2}}(0, v, 0)^T. \quad (5)$$

The Yukawa interactions which induce masses for the fermions can be written in the most general form:

$$\mathcal{L}_Y = \mathcal{L}_{\text{LNC}} + \mathcal{L}_{\text{LNV}}, \quad (6)$$

in which, each part is defined by

$$\begin{aligned} \mathcal{L}_{\text{LNC}} &= h^U \bar{Q}_{1L} \chi U_R + h_{\alpha\beta}^D \bar{Q}_{\alpha L} \chi^* D_{\beta R} \\ &\quad + h_{ab}^l \bar{\psi}_{aL} \phi l_{bR} + h_{ab}^\nu \epsilon_{pmn} (\bar{\psi}_{aL}^c)_p (\psi_{bL})_m (\phi)_n \\ &\quad + h_a^d \bar{Q}_{1L} \phi d_{aR} + h_{\alpha a}^u \bar{Q}_{\alpha L} \phi^* u_{aR} + H.c., \end{aligned} \quad (7)$$

$$\begin{aligned} \mathcal{L}_{\text{LNV}} &= s_a^u \bar{Q}_{1L} \chi u_{aR} + s_{\alpha a}^d \bar{Q}_{\alpha L} \chi^* d_{aR} \\ &\quad + s_\alpha^D \bar{Q}_{1L} \phi D_{\alpha R} + s_\alpha^U \bar{Q}_{\alpha L} \phi^* U_R + H.c., \end{aligned} \quad (8)$$

where p , m and n stand for $SU(3)_L$ indices.

The VEV ω gives mass for the exotic quarks U , D_α and the new gauge bosons Z' , X , Y , while the VEVs u and v give mass for the quarks u_a , d_a , the leptons l_a and all the ordinary gauge bosons Z , W [10]. In the next sections we will provide the detailed analysis of neutrino masses. To keep a consistency with the effective theory, the VEVs in this model have to satisfy the constraint

$$u^2 \ll v^2 \ll \omega^2. \quad (9)$$

In addition we can derive $v \approx v_{\text{weak}} = 246$ GeV and $|u| \leq 2.46$ GeV from, the mass of W boson and the ρ parameter [6], respectively. From atomic parity violation in cesium, the bound for the mass of new natural gauge boson is given by $M_{Z'} > 564$ GeV ($\omega > 1400$ GeV) [9]. From the analysis on quark masses, higher values for ω can be required, for example, up to 10 TeV [10].

The Yukawa couplings of (7) possess an extra global symmetry [12, 13] which is not broken by v, ω but by u . From these couplings, one can find the following lepton symmetry L as in Table I (only the fields with nonzero L are listed; all other fields have vanishing L). Here L is broken by u which is behind $L(\chi_1^0) = 2$, i.e., u is a kind of the *SLB scale* [14]. It is interesting that the exotic quarks also carry the lepton number (so-called lepton quarks); therefore,

TABLE I: Nonzero lepton number L of the model particles.

Field	ν_{aL}	$l_{aL,R}$	ν_{aR}^c	χ_1^0	χ_2^-	ϕ_3^+	$U_{L,R}$	$D_{\alpha L,R}$
L	1	1	-1	2	2	-2	-2	2

this L obviously does not commute with the gauge symmetry. One can then construct a new conserved charge \mathcal{L} through L by making a linear combination $L = xT_3 + yT_8 + \mathcal{L}I$. Applying L on a lepton triplet, the coefficients will be determined

$$L = \frac{4}{\sqrt{3}}T_8 + \mathcal{L}I. \quad (10)$$

Another useful conserved charge \mathcal{B} which is exactly not broken by u , v and ω is usual baryon number: $B = \mathcal{B}I$. Both the charges \mathcal{L} and \mathcal{B} for the fermion and Higgs multiplets are listed in Table II. Let us note that the Yukawa couplings

TABLE II: \mathcal{B} and \mathcal{L} charges of the model multiplets.

Multiplet	χ	ϕ	Q_{1L}	$Q_{\alpha L}$	u_{aR}	d_{aR}	U_R	$D_{\alpha R}$	ψ_{aL}	l_{aR}
\mathcal{B} -charge	0	0	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	0	0
\mathcal{L} -charge	$\frac{4}{3}$	$-\frac{2}{3}$	$-\frac{2}{3}$	$\frac{2}{3}$	0	0	-2	2	$\frac{1}{3}$	1

of (8) conserve \mathcal{B} , however, violate \mathcal{L} with ± 2 units which implies that these interactions are much smaller than the first ones [10]:

$$s_a^u, s_{\alpha a}^d, s_\alpha^D, s_\alpha^U \ll h^U, h_{\alpha\beta}^D, h_a^d, h_{\alpha a}^u. \quad (11)$$

In this model, the most general Higgs potential has very simple form

$$V(\chi, \phi) = \mu_1^2 \chi^\dagger \chi + \mu_2^2 \phi^\dagger \phi + \lambda_1 (\chi^\dagger \chi)^2 + \lambda_2 (\phi^\dagger \phi)^2 + \lambda_3 (\chi^\dagger \chi)(\phi^\dagger \phi) + \lambda_4 (\chi^\dagger \phi)(\phi^\dagger \chi). \quad (12)$$

It is noteworthy that $V(\chi, \phi)$ does not contain trilinear scalar couplings and conserves both the mentioned global symmetries, this makes the Higgs potential much simpler and discriminative from the previous ones of the 3-3-1 models [12, 13, 15]. The non-zero values of χ and ϕ at the minimum value of $V(\chi, \phi)$ can be obtained by

$$\chi^\dagger \chi = \frac{\lambda_3 \mu_2^2 - 2\lambda_2 \mu_1^2}{4\lambda_1 \lambda_2 - \lambda_3^2} \equiv \frac{u^2 + \omega^2}{2}, \quad (13)$$

$$\phi^\dagger \phi = \frac{\lambda_3 \mu_1^2 - 2\lambda_1 \mu_2^2}{4\lambda_1 \lambda_2 - \lambda_3^2} \equiv \frac{v^2}{2}. \quad (14)$$

Any other choice of u , ω for the vacuum value of χ satisfying (13) gives the same physics because it is related to (4) by an $SU(3)_L \otimes U(1)_X$ transformation. It is worth noting that the assumed $u \neq 0$ is therefore given in a general case. This model of course leads to the formation of Majoron [8, 14, 16]. The analysis in [8] shows that, after symmetry breaking, there are eight Goldstone bosons and three physical scalar fields - the SM like H^0 , the new neutral H_1^0 and the charged bilepton H_2^\pm with the masses:

$$m_H^2 \simeq \frac{4\lambda_1\lambda_2 - \lambda_3^2}{2\lambda_1}v^2, \quad m_{H_1}^2 \simeq 2\lambda_1\omega^2, \quad m_{H_2}^2 \simeq \frac{\lambda_4}{2}\omega^2. \quad (15)$$

Let us remind the reader that the couplings $\lambda_{4,1,2}$ are positive and fixed by the Higgs boson masses and the λ_3 , where the last one λ_3 is constrained by $|\lambda_3| < 2\sqrt{\lambda_1\lambda_2}$ and gives the splitting $\Delta m_H^2 \simeq -[\lambda_3^2/(2\lambda_1)]v^2$ from the SM prediction.

In the considering model, the possible different mass-mechanisms for the neutrinos can be summarized through the three dominant $SU(3)_C \otimes SU(3)_L \otimes U(1)_X$ -invariant effective operators as follows [17]:

$$O_{ab}^{\text{LNC}} = \bar{\psi}_{aL}^c \psi_{bL} \phi, \quad (16)$$

$$O_{ab}^{\text{LNV}} = (\chi^* \bar{\psi}_{aL}^c)(\chi^* \psi_{bL}), \quad (17)$$

$$O_{ab}^{\text{SLB}} = (\chi^* \bar{\psi}_{aL}^c)(\psi_{bL} \phi \chi), \quad (18)$$

where the Hermitian adjoint operators are not displayed. It is worth noting that they are also all the performable operators with the mass dimensionality $d \leq 6$ responsible for the neutrino masses. The difference among the mass-mechanisms can be verified through the operators. Both (16) and (18) conserve \mathcal{L} , while (17) violates this charge with two units. Since $d(O^{\text{LNC}}) = 4$ and $L\langle\phi\rangle = 0$, (16) provides only Dirac masses for the neutrinos which can be obtained at the tree level through the Yukawa couplings in (7). Since $d(O^{\text{SLB}}) = 6$ and $(L\langle\chi\rangle)_p \neq 0$ for $p = 1$, vanishes for other cases, (18) provides both Dirac and Majorana masses for the neutrinos through radiative corrections mediated by the model particles. The masses induced by (16) are given by the standard $SU(2)_L \otimes U(1)_Y$ symmetry breaking via the VEV v . However, those by (18) are obtained from both the stages of $SU(3)_L \otimes U(1)_X$ breaking achieved by the VEVs u , ω and v .

Let us recall that, except the unconcerned LNV couplings of (8), all the remaining interactions of the model (lepton Yukawa couplings (7), Higgs self-couplings (12), and etc.) conserve \mathcal{L} , this means that the operator (17) cannot be induced mediated by the model particles. As a fact, the economical 3-3-1 model including the alternative versions [4, 5] are only extensions beyond the SM in the scales of orders of TeV [9, 18]. Such processes are therefore expected mediated by heavy particles of an underlined new physics at a scale \mathcal{M} much greater than ω which have been followed in various of grand unified theories (GUTs) [17, 19, 20]. Thus, in this model the neutrinos can get mass from three very different sources widely ranging over the mass scales: $u \sim \mathcal{O}(1)$ GeV, $v \approx 246$ GeV, $\omega \sim \mathcal{O}(1)$ TeV, and $\mathcal{M} \sim \mathcal{O}(10^{16})$ GeV.

We remind that, in the former version [5], the authors in [21] have considered operators of the type (17), however, under a discrete symmetry [22]. As shown by us [10], the current model is realistic, and such a discrete symmetry is not needed, because, as a fact that the model will fail if it enforces. In addition, if such discrete symmetries are not discarded, the important mass contributions for the neutrinos mediated by model particles are then suppressed; for example, in this case the remaining operators (16) and (18) will be removed. With the only operator (17) the three active neutrinos will get effective zero-masses under a type II seesaw [11] (see below); however, this operator occupies a particular importance in this version.

Alternatively, in such model, the authors in [12] have examined two-loop corrections to (17) by the aid of explicit LNV Higgs self-couplings, and using a fine-tuning for the tree-level Dirac masses of (16) down to current values. However, as mentioned, this is not the case in the considering model, because our Higgs potential (12) conserves \mathcal{L} . We know that one of the problems of the 3-3-1 model with RH neutrinos is associated with the Dirac mass term of neutrinos. In the following, we will show that, if such a fine-tuning is done so that these terms get small values, then the mass generation of neutrinos mediated by model particles is not able to be done or trivially, this is in contradiction with [12]. In the next, the large bare Dirac masses for the neutrinos, which are as of charged fermions of a natural result from standard symmetry breaking, will be studied.

For the sake of convenience in further reading, we present the lepton Yukawa couplings and the relevant Higgs self-couplings in terms represented by Feynman diagrams in Figs. (1) and (2), where the Hermitian adjoint ones are not displayed.

III. NEUTRINO MASS MATRIX

The operators O^{LNC} , O^{SLB} and O^{LNV} (including their Hermitian adjoint) will provide the masses for the neutrinos: the first responsible for tree-level masses, the second for one-loop corrections, and the third for contributions of heavy particles.

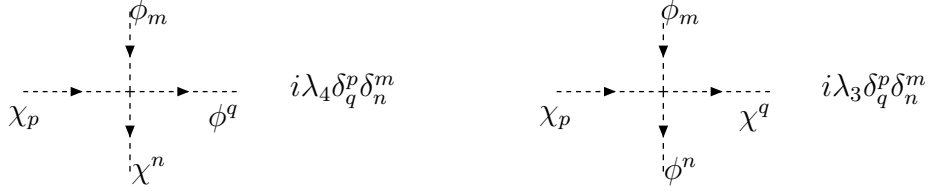


FIG. 1: Relevant Higgs self-couplings.

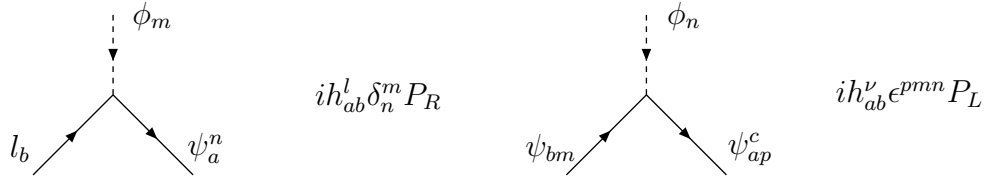


FIG. 2: Lepton Yukawa couplings.

A. Tree-level Dirac masses

From the Yukawa couplings in (7), the tree-level mass Lagrangian for the neutrinos is obtained by [23]

$$\begin{aligned}
 \mathcal{L}_{\text{mass}}^{\text{LNC}} &= h_{ab}^\nu \bar{\nu}_{aR} \nu_{bL} \langle \phi_2^0 \rangle - h_{ab}^\nu \bar{\nu}_{aL} \nu_{bR}^c \langle \phi_2^0 \rangle + H.c. \\
 &= 2 \langle \phi_2^0 \rangle h_{ab}^\nu \bar{\nu}_{aR} \nu_{bL} + H.c. = -(M_D)_{ab} \bar{\nu}_{aR} \nu_{bL} + H.c. \\
 &= -\frac{1}{2} (\bar{\nu}_{aL}^c, \bar{\nu}_{aR}) \begin{pmatrix} 0 & (M_D^T)_{ab} \\ (M_D)_{ab} & 0 \end{pmatrix} \begin{pmatrix} \nu_{bL} \\ \nu_{bR}^c \end{pmatrix} + H.c. \\
 &= -\frac{1}{2} \bar{X}_L^c M_\nu X_L + H.c.,
 \end{aligned} \tag{19}$$

where $h_{ab}^\nu = -h_{ba}^\nu$ is due to Fermi statistics. The M_D is the mass matrix for the Dirac neutrinos:

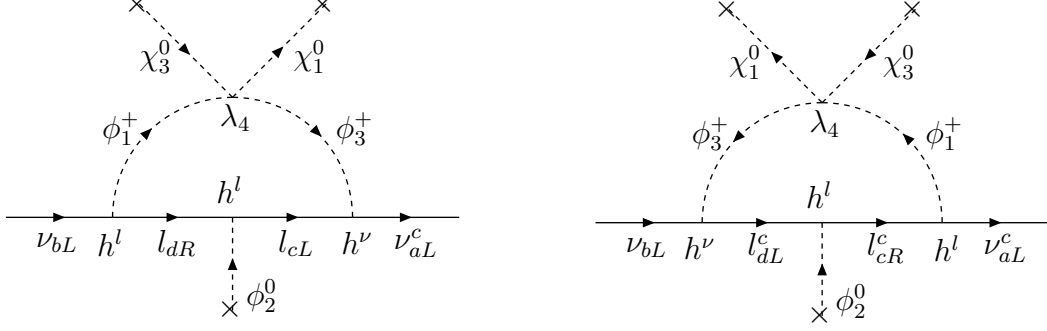
$$(M_D)_{ab} \equiv -\sqrt{2} v h_{ab}^\nu = (-M_D^T)_{ab} = \begin{pmatrix} 0 & -A & -B \\ A & 0 & -C \\ B & C & 0 \end{pmatrix}, \quad A, B, C \equiv \sqrt{2} h_{e\mu}^\nu v, \sqrt{2} h_{e\tau}^\nu v, \sqrt{2} h_{\mu\tau}^\nu v. \tag{20}$$

This mass matrix has been rewritten in a general basis $X_L^T \equiv (\nu_{eL}, \nu_{\mu L}, \nu_{\tau L}, \nu_{eR}^c, \nu_{\mu R}^c, \nu_{\tau R}^c)$:

$$M_\nu \equiv \begin{pmatrix} 0 & M_D^T \\ M_D & 0 \end{pmatrix}. \tag{21}$$

The tree-level neutrino spectrum therefore consists of only Dirac fermions. Since h_{ab}^ν is antisymmetric in a and b , the mass matrix M_D gives one neutrino massless and two others degenerate in mass: $0, -m_D, m_D$, where $m_D \equiv (A^2 + B^2 + C^2)^{1/2}$. This mass spectrum is not realistic under the data, however, it will be severely changed by the quantum corrections, the most general mass matrix can then be written as follows

$$M_\nu = \begin{pmatrix} M_L & M_D^T \\ M_D & M_R \end{pmatrix}, \tag{22}$$

FIG. 3: The one-loop corrections for the mass matrix M_L .

where $M_{L,R}$ (vanish at the tree-level) and M_D get possible corrections.

If such a tree-level contribution dominates the resulting mass matrix (after corrections), the model will provide an explanation about a large splitting either $\Delta m_{\text{atm}}^2 \gg \Delta m_{\text{sol}}^2$ or $\Delta m_{\text{LSND}}^2 \gg \Delta m_{\text{atm,sol}}^2$ [12, 24]. We then, however, must need a fine-tuning at the tree-level [12] either $m_D \sim (\Delta m_{\text{atm}}^2)^{1/2}$ ($\sim 5 \times 10^{-2}$ eV) or $m_D \sim (\Delta m_{\text{LSND}}^2)^{1/2}$ (\sim eV) [24]. Without lose the generality we can assume $h_{e\mu}^\nu \sim h_{e\tau}^\nu \sim h_{\mu\tau}^\nu$ which give us then $h^\nu \sim 10^{-13}$ (or 10^{-12}). The coupling h^ν in this case is so small and therefore this fine-tuning is not natural [25]. Indeed, as shown below, since h^ν enter the dominant corrections from (18) for $M_{L,R}$, these terms $M_{L,R}$ get very small values which are not large enough to split the degenerate neutrino masses into a realistic spectrum. (The largest degenerate splitting in squared-mass is still much smaller than $\Delta m_{\text{sol}}^2 \sim 8 \times 10^{-5}$ eV² [24].) In addition, in this case the Dirac masses get corrections trivially.

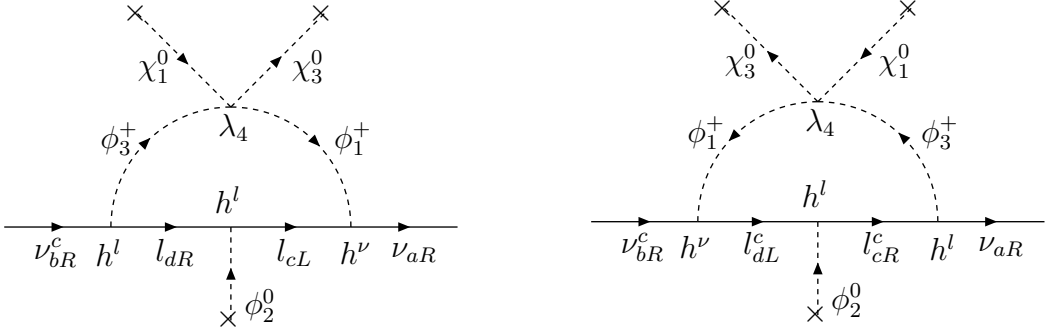
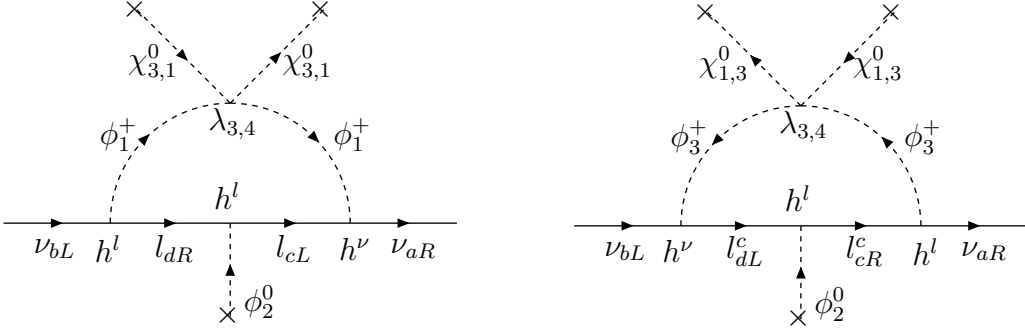
The status of this problem can be changed with the induced operator (17) (see below), however, not interested in this work, because as mentioned the operator (18) that obtains the contributions of model particles is then discarded. This implies that the tree-level Dirac mass term for the neutrinos by its naturalness should be treated as those as of the usual charged fermions resulted of the standard symmetry breaking, say, $h^\nu \sim h^e$ ($\sim 10^{-6}$) [25]. It turns out that this term is regarded as a large bare quantity and unphysical. Under the interactions, they will of course change to physical masses. In the following we will obtain such *finite renormalizations* in the masses of neutrinos.

B. One-loop level Dirac and Majorana masses

The operator (18) and its Hermitian adjoint arise from the radiative corrections mediated by the model particles, and give contributions to Majorana and Dirac mass terms M_L , M_R and M_D for the neutrinos. The Yukawa couplings of the leptons in (7) and the relevant Higgs self-couplings in (12) are explicitly rewritten as follows

$$\begin{aligned} \mathcal{L}_Y^{\text{lept}} &= 2h_{ab}^\nu \bar{\nu}_{aL}^c l_{bL} \phi_3^+ - 2h_{ab}^\nu \bar{\nu}_{aR} l_{bL} \phi_1^+ + h_{ab}^l \bar{\nu}_{aL} l_{bR} \phi_1^+ + h_{ab}^l \bar{\nu}_{aR}^c l_{bR} \phi_3^+ + H.c., \\ \mathcal{L}_H^{\text{relv}} &= \lambda_3 \phi_1^- \phi_1^+ (\chi_1^{0*} \chi_1^0 + \chi_3^{0*} \chi_3^0) + \lambda_3 \phi_3^- \phi_3^+ (\chi_1^{0*} \chi_1^0 + \chi_3^{0*} \chi_3^0) \\ &\quad + \lambda_4 \phi_1^- \phi_1^+ \chi_1^{0*} \chi_1^0 + \lambda_4 \phi_3^- \phi_3^+ \chi_3^{0*} \chi_3^0 + \lambda_4 \phi_3^- \phi_1^+ \chi_1^{0*} \chi_3^0 + \lambda_4 \phi_1^- \phi_3^+ \chi_3^{0*} \chi_1^0. \end{aligned} \quad (23)$$

The one-loop corrections to the mass matrices M_L of ν_L , M_R of ν_R and M_D of ν are therefore given in Figs. (3), (4) and (5), respectively.

FIG. 4: The one-loop corrections for the mass matrix M_R .FIG. 5: The one-loop corrections for the mass matrix M_D .

1. Radiative corrections to M_L and M_R

With the Feynman rules at hand [23], M_L is obtained by

$$\begin{aligned}
 -i(M_L)_{ab}P_L &= \int \frac{d^4p}{(2\pi)^4} (i2h_{ac}^\nu P_L) \frac{i(\not{p} + m_c)}{p^2 - m_c^2} \left(ih_{cd}^l \frac{v}{\sqrt{2}} P_R \right) \frac{i(\not{p} + m_d)}{p^2 - m_d^2} \\
 &\times (ih_{bd}^{l*} P_L) \frac{-1}{(p^2 - m_{\phi_1}^2)(p^2 - m_{\phi_3}^2)} \left(i\lambda_4 \frac{u\omega}{2} \right) \\
 &+ \int \frac{d^4p}{(2\pi)^4} (ih_{ac}^{l*} P_L) \frac{i(-\not{p} + m_c)}{p^2 - m_c^2} \left(ih_{dc}^l \frac{v}{\sqrt{2}} P_R \right) \frac{i(-\not{p} + m_d)}{p^2 - m_d^2} \\
 &\times (i2h_{bd}^\nu P_L) \frac{-1}{(p^2 - m_{\phi_1}^2)(p^2 - m_{\phi_3}^2)} \left(i\lambda_4 \frac{u\omega}{2} \right). \tag{24}
 \end{aligned}$$

For the sake of simplicity, in the following, we suppose that the Yukawa coupling of charged leptons h^l is flavor diagonal, thus l_c and l_d are mass eigenstates respective to the mass eigenvalues m_c and m_d . The equation (24) becomes then

$$(M_L)_{ab} = \frac{i\sqrt{2}\lambda_4 u\omega}{v} h_{ab}^\nu [m_b^2 I(m_b^2, m_{\phi_3}^2, m_{\phi_1}^2) - m_a^2 I(m_a^2, m_{\phi_3}^2, m_{\phi_1}^2)], \quad (a, b \text{ not summed}), \tag{25}$$

where the integral $I(a, b, c)$ is given in Appendix A.

In the effective approximation (9), identifications are given by $\phi_3^\pm \sim H_2^\pm$ and $\phi_1^\pm \sim G_W^\pm$ [8], where H_2^\pm and G_W^\pm as above mentioned, are the charged bilepton Higgs boson and the Goldstone boson associated with W^\pm boson,

respectively. For the masses, we have also $m_{\phi_3}^2 \simeq m_{H_2}^2 (\simeq \frac{\lambda_4}{2}\omega^2)$ and $m_{\phi_1}^2 \simeq 0$. Using (A5), the integrals are given by

$$I(m_a^2, m_{\phi_3}^2, m_{\phi_1}^2) \simeq -\frac{i}{16\pi^2} \frac{1}{m_a^2 - m_{H_2}^2} \left[1 - \frac{m_{H_2}^2}{m_a^2 - m_{H_2}^2} \ln \frac{m_a^2}{m_{H_2}^2} \right], \quad a = e, \mu, \tau. \quad (26)$$

Consequently, the mass matrix (25) becomes

$$\begin{aligned} (M_L)_{ab} &\simeq \frac{\sqrt{2}\lambda_4 u \omega h_{ab}^\nu}{16\pi^2 v} \left[\frac{m_{H_2}^2(m_a^2 - m_b^2)}{(m_b^2 - m_{H_2}^2)(m_a^2 - m_{H_2}^2)} + \frac{m_a^2 m_{H_2}^2}{(m_a^2 - m_{H_2}^2)^2} \ln \frac{m_a^2}{m_{H_2}^2} - \frac{m_b^2 m_{H_2}^2}{(m_b^2 - m_{H_2}^2)^2} \ln \frac{m_b^2}{m_{H_2}^2} \right] \\ &\simeq \frac{\sqrt{2}\lambda_4 u \omega h_{ab}^\nu}{16\pi^2 v m_{H_2}^2} \left[m_a^2 \left(1 + \ln \frac{m_a^2}{m_{H_2}^2} \right) - m_b^2 \left(1 + \ln \frac{m_b^2}{m_{H_2}^2} \right) \right], \end{aligned} \quad (27)$$

where the last approximation (27) is kept in the orders up to $\mathcal{O}[(m_{a,b}^2/m_{H_2}^2)^2]$. Since $m_{H_2}^2 \simeq \frac{\lambda_4}{2}\omega^2$, it is worth noting that the resulting M_L is not explicitly dependent on λ_4 , however, proportional to $t_\theta = u/\omega$ (the mixing angle between the W boson and the singly-charged bilepton gauge boson Y [6]), $\sqrt{2}v h_{ab}^\nu$ (the tree-level Dirac mass term of neutrinos), and m_{H_2} in the logarithm scale. Here the VEV $v \approx v_{\text{weak}}$, and the charged-lepton masses m_a ($a = e, \mu, \tau$) have the well-known values. Let us note that M_L is symmetric and has vanishing diagonal elements.

For the corrections to M_R , it is easily to check that the relationship $(M_R)_{ab} = -(M_L)_{ab}$ is exact at the one-loop level. (This result can be derived from Fig. (4) in a general case without imposing any additional condition on h^l , h^ν , and further.) Combining this result with (27), the mass matrices are explicitly rewritten as follows

$$(M_L)_{ab} = -(M_R)_{ab} \simeq \begin{pmatrix} 0 & f & r \\ f & 0 & t \\ r & t & 0 \end{pmatrix}, \quad (28)$$

where the elements are obtained by

$$\begin{aligned} f &\equiv \left(\sqrt{2}v h_{e\mu}^\nu \right) \left\{ \left(\frac{t_\theta}{8\pi^2 v^2} \right) \left[m_e^2 \left(1 + \ln \frac{m_e^2}{m_{H_2}^2} \right) - m_\mu^2 \left(1 + \ln \frac{m_\mu^2}{m_{H_2}^2} \right) \right] \right\}, \\ r &\equiv \left(\sqrt{2}v h_{e\tau}^\nu \right) \left\{ \left(\frac{t_\theta}{8\pi^2 v^2} \right) \left[m_e^2 \left(1 + \ln \frac{m_e^2}{m_{H_2}^2} \right) - m_\tau^2 \left(1 + \ln \frac{m_\tau^2}{m_{H_2}^2} \right) \right] \right\}, \\ t &\equiv \left(\sqrt{2}v h_{\mu\tau}^\nu \right) \left\{ \left(\frac{t_\theta}{8\pi^2 v^2} \right) \left[m_\mu^2 \left(1 + \ln \frac{m_\mu^2}{m_{H_2}^2} \right) - m_\tau^2 \left(1 + \ln \frac{m_\tau^2}{m_{H_2}^2} \right) \right] \right\}. \end{aligned} \quad (29)$$

It can be checked that f, r, t are much smaller than those of M_D . To see this, we can take $m_e \simeq 0.51099$ MeV, $m_\mu \simeq 105.65835$ MeV, $m_\tau \simeq 1777$ MeV, $v \simeq 246$ GeV, $u \simeq 2.46$ GeV, $\omega \simeq 3000$ GeV, and $m_{H_2} \simeq 700$ GeV ($\lambda_4 \sim 0.11$) [6, 8, 9], which give us then

$$f \simeq \left(\sqrt{2}v h_{e\mu}^\nu \right) (3.18 \times 10^{-11}), \quad r \simeq \left(\sqrt{2}v h_{e\tau}^\nu \right) (5.93 \times 10^{-9}), \quad t \simeq \left(\sqrt{2}v h_{\mu\tau}^\nu \right) (5.90 \times 10^{-9}), \quad (30)$$

where the second factors rescale negligibly with $\omega \sim 1 - 10$ TeV and $m_{H_2} \sim 200 - 2000$ GeV. This thus implies that

$$|M_{L,R}|/|M_D| \sim 10^{-9}, \quad (31)$$

which can be checked with the help of $|M| \equiv (M^+ M)^{1/2}$. In other words, the constraint is given as follows

$$|M_{L,R}| \ll |M_D|. \quad (32)$$

With the above results at hand, we can then get the masses by studying diagonalization of the mass matrix (22), in which, the submatrices $M_{L,R}$ and M_D satisfying the constraint (32), are given by (28) and (20), respectively. In calculation, let us note that, since M_D has one vanishing eigenvalue, M_ν does not possess the pseudo-Dirac property in all three generations [26], however, is very close to those because the remaining eigenvalues do. As a fact, we will see that M_ν contains a combined framework of the seesaw [11] and the pseudo-Dirac [27]. To get mass, we can suppose that h^ν is real, and therefore the matrix iM_D is Hermitian: $(iM_D)^+ = iM_D$ (20). The Hermitianity for $M_{L,R}$ is also followed by (28). Because the dominant matrix is M_D (32), we first diagonalize it by biunitary transformation [28]:

$$\bar{\nu}_{aR} = \bar{\nu}_{iR}(-iU)_{ia}^+, \quad \nu_{bL} = U_{bj}\nu_{jL}, \quad (i, j = 1, 2, 3), \quad (33)$$

$$M_{\text{diag}} \equiv \text{diag}(0, -m_D, m_D) = (-iU)^+ M_D U, \quad m_D = \sqrt{A^2 + B^2 + C^2}, \quad (34)$$

where the matrix U is easily obtained by

$$U = \frac{1}{m_D \sqrt{2(A^2 + C^2)}} \begin{pmatrix} C \sqrt{2(A^2 + C^2)} & iBC - Am_D & BC - iAm_D \\ -B \sqrt{2(A^2 + C^2)} & i(A^2 + C^2) & (A^2 + C^2) \\ A \sqrt{2(A^2 + C^2)} & iAB + Cm_D & AB + iCm_D \end{pmatrix}. \quad (35)$$

Resulted by the anti-Hermiticity of M_D , it is worth noting that M_ν in the case of vanishing $M_{L,R}$ (21) is indeed diagonalized by the following unitary transformation:

$$V = \frac{1}{\sqrt{2}} \begin{pmatrix} U & U \\ -iU & iU \end{pmatrix}. \quad (36)$$

A new basis $(\nu_1, \nu_2, \dots, \nu_6)_L^T \equiv V^+ X_L^T$, which is different from $(\nu_{jL}, \nu_{iR}^c)^T$ of (33), is therefore performed. The neutrino mass matrix (22) in this basis becomes

$$V^+ M_\nu V = \begin{pmatrix} M_{\text{diag}} & \epsilon \\ \epsilon & -M_{\text{diag}} \end{pmatrix}, \quad (37)$$

$$\epsilon \equiv U^+ M_L U, \quad \epsilon^+ = \epsilon, \quad (38)$$

where the elements of ϵ are obtained by

$$\epsilon_{11} = \epsilon_{22} = \epsilon_{33} = 0, \quad (39)$$

$$\begin{aligned} \epsilon_{12} = i\epsilon_{13}^* = & \{ [ABm_D + iC(A^2 - B^2 + C^2)]f + [(C^2 - A^2)m_D + 2iABC]r \\ & + [iA(A^2 - B^2 + C^2) - BCm_D]t \} [m_D^2 \sqrt{2(A^2 + C^2)}]^{-1}, \end{aligned} \quad (40)$$

$$\begin{aligned} \epsilon_{23} = & \{ (A^2 + C^2) [(Cm_D - iAB)t - (Am_D + iBC)f] \\ & - [B(A^2 - C^2)m_D + iAC(A^2 + 2B^2 + C^2)]r \} [m_D^2(A^2 + C^2)]^{-1}. \end{aligned} \quad (41)$$

Let us remind the reader that (39) is exactly given at the one-loop level M_L (25) without imposing any approximation on this mass matrix. Interchanging the positions of component fields in the basis $(\nu_1, \nu_2, \dots, \nu_6)_L^T$ by a permutation transformation $P^+ \equiv P_{23}P_{34}$, that is, $\nu_p \rightarrow (P^+)_{pq}\nu_q$ ($p, q = 1, 2, \dots, 6$) with

$$P^+ = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \quad (42)$$

the mass matrix (37) can be rewritten as follows

$$P^+(V^+ M_\nu V)P = \left(\begin{array}{cc|cccc} 0 & 0 & 0 & 0 & \epsilon_{12} & \epsilon_{13} \\ 0 & 0 & \epsilon_{12} & \epsilon_{13} & 0 & 0 \\ \hline 0 & \epsilon_{21} & -m_D & 0 & 0 & \epsilon_{23} \\ 0 & \epsilon_{31} & 0 & m_D & \epsilon_{32} & 0 \\ \hline \epsilon_{21} & 0 & 0 & \epsilon_{23} & m_D & 0 \\ \epsilon_{31} & 0 & \epsilon_{32} & 0 & 0 & -m_D \end{array} \right). \quad (43)$$

It is worth noting that in (43) all the off-diagonal components $|\epsilon|$ are much smaller than the eigenvalues $|\pm m_D|$ due to the condition (32). The degenerate eigenvalues 0, $-m_D$ and $+m_D$ (each twice) are now splitting into three pairs with six different values, two light and four heavy. The two neutrinos of first pair resulted by the 0 splitting have very small masses as a result of exactly what a seesaw does [11], that is, the off-diagonal block contributions to these masses are suppressed by the large pseudo-Dirac masses of the lower-right block. The suppression in this case is different from the usual ones [11] because it needs only the pseudo-Dirac particles [27] with the masses m_D of the electroweak scale instead of extremely heavy RH Majorana fields, and that the Dirac masses in those mechanisms are now played by loop-induced f, r, t (29) as a result of the SLB u/ω . Therefore, the mass matrix (43) is effectively decomposed into M_S for the first pair of light neutrinos (ν_S) and M_P for the last two pairs of heavy pseudo-Dirac neutrinos (ν_P):

$$(\nu_1, \nu_4, \nu_2, \nu_3, \nu_5, \nu_6)_L^T \rightarrow (\nu_S, \nu_P)_L^T = V_{\text{eff}}^+(\nu_1, \nu_4, \nu_2, \nu_3, \nu_5, \nu_6)_L^T, \quad V_{\text{eff}}^+(P^+ V^+ M_\nu V P) V_{\text{eff}} = \text{diag}(M_S, M_P), \quad (44)$$

where V_{eff} , M_S and M_P get the approximations:

$$V_{\text{eff}} \simeq \begin{pmatrix} 1 & \mathcal{E} \\ -\mathcal{E}^+ & 1 \end{pmatrix}, \quad \mathcal{E} \equiv \begin{pmatrix} 0 & 0 & \epsilon_{12} & \epsilon_{13} \\ \epsilon_{12} & \epsilon_{13} & 0 & 0 \end{pmatrix} \begin{pmatrix} -m_D & 0 & 0 & \epsilon_{23} \\ 0 & m_D & \epsilon_{32} & 0 \\ 0 & \epsilon_{23} & m_D & 0 \\ \epsilon_{32} & 0 & 0 & -m_D \end{pmatrix}^{-1},$$

$$M_S \simeq -\mathcal{E} \begin{pmatrix} 0 & \epsilon_{21} \\ 0 & \epsilon_{31} \\ \epsilon_{21} & 0 \\ \epsilon_{31} & 0 \end{pmatrix}, \quad M_P \simeq \begin{pmatrix} -m_D & 0 & 0 & \epsilon_{23} \\ 0 & m_D & \epsilon_{32} & 0 \\ 0 & \epsilon_{23} & m_D & 0 \\ \epsilon_{32} & 0 & 0 & -m_D \end{pmatrix}. \quad (45)$$

The mass matrices M_S and M_P , respectively, give exact solutions as follows

$$m_{S\pm} = \pm \frac{2\text{Im}(\epsilon_{13}\epsilon_{13}\epsilon_{32})}{m_D^2 - \epsilon_{23}^2} \simeq \pm 2\text{Im}\left(\frac{\epsilon_{13}\epsilon_{13}\epsilon_{32}}{m_D^2}\right), \quad (46)$$

$$m_{P\pm} = -m_D \pm |\epsilon_{23}|, \quad m_{P'\pm} = m_D \pm |\epsilon_{23}|. \quad (47)$$

In this case, the mixing matrices are collected into $(\nu_{S\pm}, \nu_{P\pm}, \nu_{P'\pm})_L^T = V_{\pm}^+(\nu_S, \nu_P)_L^T$, where the V_{\pm} is obtained by

$$V_{\pm} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & \kappa & -\kappa & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & \kappa & -\kappa \\ 0 & 0 & 1 & 1 & 0 & 0 \end{pmatrix}, \quad \kappa \equiv \frac{\epsilon_{23}}{|\epsilon_{23}|} = \exp(i \arg \epsilon_{23}). \quad (48)$$

It is noted that the degeneration in the Dirac one $|\pm m_D|$ is now splitting severally.

From (47) we see that the four large pseudo-Dirac masses for the neutrinos are almost degenerate. In addition, the resulting spectrum (46), (47) yields two largest squared-mass splittings, respectively, proportional to m_D^2 and $4m_D|\epsilon_{23}|$. Resulted by (41) and (30), we can evaluate $|\epsilon_{23}| \simeq 3.95 \times 10^{-9} m_D \ll m_D$ (where $A \sim B \sim C \sim m_D/\sqrt{3}$ is understood), this therefore implies that the fine-tuning as mentioned is not realistic because the splitting $4m_D|\epsilon_{23}|$ is still much smaller than Δm_{sol}^2 . (In detail, in Table III, we give the numerical values of these fine-tunings, where the parameters are given as before (30).)

TABLE III: The values for h^ν , elements of M_ν , and two largest splittings in squared-mass.

Fine-tuning	h^ν	$\sqrt{2}vh^\nu$ (eV)	f (eV)	r (eV)	t (eV)	m_D^2 (eV ²)	$4m_D \epsilon_{23} $ (eV ²)
$m_D^2 \sim \Delta m_{\text{atm}}^2$	8.30×10^{-14}	2.88×10^{-2}	9.18×10^{-13}	1.71×10^{-10}	1.70×10^{-10}	2.50×10^{-3}	3.95×10^{-11}
$m_D^2 \sim \Delta m_{\text{LSND}}^2$	1.66×10^{-12}	5.77×10^{-1}	1.83×10^{-11}	3.43×10^{-9}	3.41×10^{-9}	1.00	1.58×10^{-8}

Similarly, for the two small masses, we can also evaluate $|m_{S\pm}| \simeq 4.29 \times 10^{-28} m_D$. This shows that the masses $m_{S\pm}$ are very much smaller than the splitting $|\epsilon_{23}|$. This also implies that the two light neutrinos in this case are hidden for any m_D value of pseudo-Dirac neutrinos. Let us see the sources of the problem why these masses are so small: (i) All the elements of left-upper block of (43) of the two neutrinos vanish; (ii) In (46) the resulting masses are proportional to $|\epsilon|^3/m_D^2$, but not to $|\epsilon|^2/m_D$ as expected from (43). It turns out that this is due to the antisymmetric of h_{ab}^ν enforcing on the tree-level Dirac-mass matrix and the degenerate of $M_R = -M_L$ of the one-loop level left-handed (LH) and RH Majorana-mass matrices. It can be easily checked that such degeneration in Majorana masses is remained up to higher-order radiative corrections as a result of treating the LH and RH neutrinos in the same gauge triplets with the model Higgs content; for example, by the aid of (18) the degeneration is given up to any higher-order loop.

2. Radiative corrections to M_D

As mentioned, the mass matrix M_D requires the one-loop corrections as given in Fig. 5, and the contributions are easily obtained as follows

$$\begin{aligned}
-i(M_D^{\text{rad}})_{ab}P_L &= \int \frac{d^4p}{(2\pi)^4} (-i2h_{ac}^\nu P_L) \frac{i(\not{p} + m_c)}{p^2 - m_c^2} \left(ih_{cd}^l \frac{v}{\sqrt{2}} P_R \right) \frac{i(\not{p} + m_d)}{p^2 - m_d^2} \\
&\times (ih_{bd}^{l*} P_L) \frac{-1}{(p^2 - m_{\phi_1}^2)^2} \left(i\lambda_3 \frac{u^2 + \omega^2}{2} + i\lambda_4 \frac{u^2}{2} \right) \\
&+ \int \frac{d^4p}{(2\pi)^4} (ih_{ac}^{l*} P_L) \frac{i(-\not{p} + m_c)}{p^2 - m_c^2} \left(ih_{dc}^l \frac{v}{\sqrt{2}} P_R \right) \frac{i(-\not{p} + m_d)}{p^2 - m_d^2} \\
&\times (i2h_{bd}^\nu P_L) \frac{-1}{(p^2 - m_{\phi_3}^2)^2} \left(i\lambda_3 \frac{u^2 + \omega^2}{2} + i\lambda_4 \frac{\omega^2}{2} \right). \tag{49}
\end{aligned}$$

We rewrite

$$\begin{aligned}
(M_D^{\text{rad}})_{ab} &= -\frac{i\sqrt{2}h_{ab}^\nu}{v} \left\{ [\lambda_3(u^2 + \omega^2) + \lambda_4 u^2] m_b^2 I(m_b^2, m_{\phi_1}^2) \right. \\
&\quad \left. + [\lambda_3(u^2 + \omega^2) + \lambda_4 \omega^2] m_a^2 I(m_a^2, m_{\phi_3}^2) \right\}, \quad (a, b \text{ not summed}), \tag{50}
\end{aligned}$$

where $I(a, b)$ is given in (A6). With the help of (A7), the approximation for (50) is obtained by

$$\begin{aligned}
(M_D^{\text{rad}})_{ab} &\simeq -\frac{h_{ab}^\nu}{8\sqrt{2}\pi^2 v} \left\{ [\lambda_3(u^2 + \omega^2) + \lambda_4 u^2] + [\lambda_3(u^2 + \omega^2) + \lambda_4 \omega^2] \frac{m_a^2}{m_{H_2}^2} \right\} \\
&= -\sqrt{2}h_{ab}^\nu \left(\frac{\lambda_3 \omega^2}{16\pi^2 v} \right) \left[1 + \left(1 + \frac{\lambda_4}{\lambda_3} \right) \left(\frac{u^2}{\omega^2} + \frac{m_a^2}{m_{H_2}^2} \right) + \mathcal{O} \left(\frac{u^4}{\omega^4}, \frac{m_{a,b}^4}{m_{H_2}^4} \right) \right]. \tag{51}
\end{aligned}$$

Because of the constraint (9) the higher-order corrections $\mathcal{O}(\dots)$ can be neglected, thus M_D^{rad} is rewritten as follows

$$(M_D^{\text{rad}})_{ab} = -\sqrt{2}h_{ab}^\nu \left(\frac{\lambda_3 \omega^2}{16\pi^2 v} \right) (1 + \delta_a), \quad \delta_a \equiv \left(1 + \frac{\lambda_4}{\lambda_3} \right) \left(\frac{u^2}{\omega^2} + \frac{m_a^2}{m_{H_2}^2} \right), \tag{52}$$

where δ_a is of course an infinitesimal coefficient, i.e., $|\delta| \ll 1$. Again, this implies also that if the fine-tuning is done the resulting Dirac-mass matrix get trivially. It is due to the fact that the contribution of the term associated with δ_a in (52) is then very small and neglected, the remaining term gives an antisymmetric resulting Dirac-mass matrix, that is therefore unrealistic under the data.

With this result, it is worth noting that the scale

$$\left| \frac{\lambda_3 \omega^2}{16\pi^2 v} \right| \tag{53}$$

of the radiative Dirac masses (52) is in the orders of the scale v of the tree-level Dirac masses (20). Indeed, if one puts $|(\lambda_3 \omega^2)/(16\pi^2 v)| = v$ and takes $|\lambda_3| \sim 0.1 - 1$, then $\omega \sim 3 - 10$ TeV as expected in the constraints [9, 18]. The resulting Dirac-mass matrix which is combined of (20) and (52) therefore gets two typical examples of the bounds: (i) $(\lambda_3 \omega^2)/(16\pi^2 v) + v \sim \mathcal{O}(v)$; (ii) $(\lambda_3 \omega^2)/(16\pi^2 v) + v \sim \mathcal{O}(0)$. The first case (i) yields that the status on the masses of neutrinos as given above is remained unchanged and therefore is also trivial as mentioned. In the last case (ii), the combination of (20) and (52) gives

$$(M_D)_{ab} = \sqrt{2}h_{ab}^\nu (v\delta_a). \tag{54}$$

It is interesting that in this case the scale v for the Dirac masses (20) gets naturally a large reduction, and we argue that this is not a fine-tuning. Because the large radiative mass term in (52) is canceled by the tree-level Dirac masses, we mean this as a finite renormalization in the masses of neutrinos. It is also noteworthy that, unlike the case of the tree-level mass term (20), the mass matrix (54) is now *nonantisymmetric in a and b* . Thus all the three eigenvalues of this matrix are nonzero. Let us recall that in the first case (i) the zero eigenvalue is, however, retained because the combination of (20) and (52) is proportional to $h_{ab}^\nu v$.

In contrast to (31), in this case there is no large hierarchy between $M_{L,R}$ and M_D . To see this explicitly, let us take the values of the parameters as given before (30), thus $\lambda_3 \simeq -1.06$ and the coefficients δ_a are evaluated by

$$\delta_e \simeq 6.03 \times 10^{-7}, \quad \delta_\mu \simeq 6.23 \times 10^{-7}, \quad \delta_\tau \simeq 6.28 \times 10^{-6}. \quad (55)$$

Hence, we get

$$|M_{L,R}|/|M_D| \sim 10^{-2} - 10^{-3}. \quad (56)$$

With the values given in (55), the quantities h^ν and m_D can be evaluated through the mass term (54); the neutrino data imply that h^ν and m_D are *in the orders of* h^e and m_e - the Yukawa coupling and mass of electron, respectively. It is worth checking that the two largest squared-mass splittings as given before can be applied on this case of (56), and seeing that they fit *naturally* the data.

C. Mass contributions from heavy particles

There remain now two questions not yet answered: (i) The degeneration of $M_R = -M_L$; (ii) The hierarchy of $M_{L,R}$ and M_D (56) can be continuously reduced? As mentioned, we will prove that the new physics gives us the solution.

The mass Lagrangian for the neutrinos given by the operator (17) can be explicitly written as follows

$$\begin{aligned} \mathcal{L}_{\text{mass}}^{\text{LNV}} &= s_{ab}^\nu \mathcal{M}^{-1} (\langle \chi^* \rangle \bar{\psi}_{aL}^c) (\langle \chi^* \rangle \psi_{bL}) + H.c. \\ &= s_{ab}^\nu \mathcal{M}^{-1} \left(\frac{u}{\sqrt{2}} \bar{\nu}_{aL}^c + \frac{\omega}{\sqrt{2}} \bar{\nu}_{aR} \right) \left(\frac{u}{\sqrt{2}} \nu_{bL} + \frac{\omega}{\sqrt{2}} \nu_{bR}^c \right) + H.c. \\ &= -\frac{1}{2} \bar{X}_L^c M_\nu^{\text{new}} X_L + H.c., \end{aligned} \quad (57)$$

where the mass matrix for the neutrinos is obtained by

$$M_\nu^{\text{new}} \equiv - \begin{pmatrix} \frac{u^2}{\mathcal{M}} s^\nu & \frac{u\omega}{\mathcal{M}} s^\nu \\ \frac{u\omega}{\mathcal{M}} s^\nu & \frac{\omega^2}{\mathcal{M}} s^\nu \end{pmatrix}, \quad (58)$$

in which, the coupling s_{ab}^ν is symmetric in a and b . Because of the hierarchies $u^2 \ll u\omega \ll \omega^2$ corresponding of the submatrices M_L^{new} , M_D^{new} and M_R^{new} of (58), the two questions as mentioned are *solved simultaneously*. Intriguing comparisons between s^ν and h^ν are given in order

1. h^ν conserves the lepton number; s^ν violates this charge.
2. h^ν is antisymmetric and enforcing on the Dirac-mass matrix; s^ν is symmetric and breaks this property.
3. h^ν preserves the degeneration of $M_R = -M_L$; s^ν breaks the $M_R = -M_L$.
4. A pair of (s^ν, h^ν) in the lepton sector will complete the rule played by the quark couplings (s^q, h^q) [10].
5. h^ν defines the interactions in the SM scale v ; s^ν gives those in the GUT scale \mathcal{M} .

Let us now take the values $\mathcal{M} \simeq 10^{16}$ GeV, $\omega \simeq 3000$ GeV, $u \simeq 2.46$ GeV and $s^\nu \sim \mathcal{O}(1)$ (perhaps smaller), the submatrices $M_L^{\text{new}} \simeq -6.05 \times 10^{-7} s^\nu$ eV and $M_D^{\text{new}} \simeq -7.38 \times 10^{-4} s^\nu$ eV can give contributions (to the diagonal components of M_L and M_D , respectively) but very small. It is noteworthy that the last one $M_R^{\text{new}} \simeq -0.9 s^\nu$ eV can *dominate* M_R .

To summarize, in this model the neutrino mass matrix is combined by $M_\nu + M_\nu^{\text{new}}$ where the first term is defined by (22), and the last term by (58); the submatrices of M_ν are given in (28) and (54), respectively. Dependence on the strength of the new physics coupling s^ν , the submatrices of the last term, M_L^{new} and M_D^{new} , are included or removed.

IV. CONCLUSIONS

The basic motivation of this work is to study neutrino mass in the framework of the model based on $\text{SU}(3)_C \otimes \text{SU}(3)_L \otimes \text{U}(1)_X$ gauge group contained minimal Higgs sector with right-handed neutrinos. In this paper, the masses of neutrinos are given by three different sources widely ranging over the mass scales including the GUT's and the

small VEV u of spontaneous lepton breaking. We have shown that, at the tree-level, there are three Dirac neutrinos: one massless and two degenerate with the masses in the order of the electron mass. At the one-loop level, a possible framework for the finite renormalization of the neutrino masses is obtained. The Dirac masses obtain a large reduction, the Majorana mass types get degenerate in $M_R = -M_L$, all these masses are in the bound of the data. It is emphasized that the above degeneration is a consequence of the fact that the left-handed and right-handed neutrinos, in this model, are in the same gauge triplets. The new physics including the 3-3-1 model are strongly signified. The degenerations and the hierarchies among the mass types are completely removed by heavy particles.

We have shown that the three couplings $\lambda_{1,2,4}$ of the Higgs potential are constrained by the scalar masses, the remainder λ_3 is free, negative [8] and now fixed by the neutrino masses. This means also that the generation of the neutrino masses leads to a shift (down) in mass of the Higgs boson from the SM prediction. In this model, the Goldstone boson $G_X \sim \chi_1^0$ of the non-Hermitian neutral bilepton gauge boson X^0 [8] is also a Majoron associated with the neutrino Majorana masses. The coupling of $HG_X G_X$ is given at the tree-level [8] and will provide a considerable contribution in the invisible modes of decay of the SM Higgs boson.

The resulting mass matrix for the neutrinos consists of two parts $M_\nu + M_\nu^{\text{new}}$: the first is mediated by the model particles, and the last is due to the new physics. Upon the contributions of M_ν^{new} , the different realistic mass textures can be produced such as pseudo-Dirac patterns associated with the seesaw one are obtained in case of the last term hidden (neglected). In another scenario, that the bare coupling h^ν of Dirac masses get higher values, for example, in orders of $h^{\mu,\tau}$, the VEV ω can be picked up to an enough large value ($\sim \mathcal{O}(10^4 - 10^5)$ TeV) so that the type II seesaw spectrum is obtained. Such features deserves further study.

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APPENDIX A: FEYNMAN INTEGRATION

In this Appendix, we present evaluation of the integral

$$I(a, b, c) \equiv \int \frac{d^4 p}{(2\pi)^4} \frac{p^2}{(p^2 - a)^2 (p^2 - b)(p^2 - c)}, \quad (\text{A1})$$

where $a, b, c > 0$ and $I(a, b, c) = I(a, c, b)$ should be noted in use.

In the case of $b \neq c$ and $b, c \neq a$, we introduce a well-known integral as follows

$$\int \frac{d^4 p}{(2\pi)^4} \frac{1}{(p^2 - a)(p^2 - b)(p^2 - c)} = \frac{-i}{16\pi^2} \left\{ \frac{a \ln a}{(a - b)(a - c)} + \frac{b \ln b}{(b - a)(b - c)} + \frac{c \ln c}{(c - b)(c - a)} \right\}. \quad (\text{A2})$$

Differentiating two sides of this equation with respect to a we have

$$\begin{aligned} \int \frac{d^4 p}{(2\pi)^4} \frac{1}{(p^2 - a)^2 (p^2 - b)(p^2 - c)} &= \frac{-i}{16\pi^2} \left\{ \frac{\ln a + 1}{(a - b)(a - c)} - \frac{a(2a - b - c) \ln a}{(a - b)^2 (a - c)^2} \right. \\ &\quad \left. + \frac{b \ln b}{(b - a)^2 (b - c)} + \frac{c \ln c}{(c - a)^2 (c - b)} \right\}. \end{aligned} \quad (\text{A3})$$

Combining (A2) and (A3) the integral (A1) becomes

$$\begin{aligned} I(a, b, c) &= \int \frac{d^4 p}{(2\pi)^4} \left[\frac{1}{(p^2 - a)(p^2 - b)(p^2 - c)} + \frac{a}{(p^2 - a)^2 (p^2 - b)(p^2 - c)} \right] \\ &= \frac{-i}{16\pi^2} \left\{ \frac{a(2 \ln a + 1)}{(a - b)(a - c)} - \frac{a^2(2a - b - c) \ln a}{(a - b)^2 (a - c)^2} + \frac{b^2 \ln b}{(b - a)^2 (b - c)} + \frac{c^2 \ln c}{(c - a)^2 (c - b)} \right\}. \end{aligned} \quad (\text{A4})$$

If $a, b \gg c$ or $c \simeq 0$, we have an approximation as follows

$$I(a, b, c) \simeq -\frac{i}{16\pi^2} \frac{1}{a-b} \left[1 - \frac{b}{a-b} \ln \frac{a}{b} \right]. \quad (\text{A5})$$

In the other case with $b = c$ and $b \neq a$, we have also

$$I(a, b) \equiv I(a, b, b) = -\frac{i}{16\pi^2} \left[\frac{a+b}{(a-b)^2} - \frac{2ab}{(a-b)^3} \ln \frac{a}{b} \right], \quad (\text{A6})$$

where, also, $I(a, b) = I(b, a)$ should be noted in use.

If $b \gg a$ or $a \simeq 0$, we have the following approximation

$$I(a, b) \simeq -\frac{i}{16\pi^2 b}. \quad (\text{A7})$$

Let us note that the above approximations $aI(a, b, c)$ (or $bI(a, b, c)$) and $bI(a, b)$ are kept in the orders up to $\mathcal{O}(c/a, c/b)$ and $\mathcal{O}(a/b)$, respectively.

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